Objective To choose the best method for solving a system of linear equations

Essential Understanding You can solve systems of linear equations using a graph, the substitution method, or the elimination method. The best method to use depends on the forms of the given equations and how precise the solution should be.

Lesson Check

Do you know HOW?

1. **Newsletters** Printing a newsletter costs $1.50 per copy plus $450 in printer's fees. The copies are sold for $3 each. How many copies of the newsletter must be sold to break even?

2. **Jewelry** A metal alloy is a metal made by blending 2 or more types of metal. A jeweler has supplies of two metal alloys. One alloy is 30% gold and the other is 10% gold. How much of each alloy should the jeweler combine to create 4 kg of an alloy containing 15% gold?

3. **Flying** With a tailwind, a bird flew at a ground speed of 3 mi/h. Flying the same path against the same wind, the bird travels at a ground speed of 1.5 mi/h. What is the bird's air speed? What is the wind speed?

1. 300 copies

2. 1 kg of 30% gold, 3 kg of 10% gold

3. 2.25 mi/h; 0.75 mi/h

Do you UNDERSTAND? MATHEMATICAL PRACTICES

4. Vocabulary What is the relationship between income and expenses before a break-even point is reached? What is the relationship between income and expenses after a break-even point is reached?

5. Reasoning Which method would you use to solve the following system? Explain.

$$3x + 2y = 9$$
$$-2x + 3y = 5$$

6. Reasoning One brand of cranberry-apple drink is 15% cranberry juice. Another brand is 40% cranberry juice. You would like to combine the brands to make a drink that is 25% cranberry juice. Without calculating, which brand of juice will you need more of to make your drink? Explain.

4. Before the break-even point, expenses exceed income. After the break-even point, income exceeds expenses.

5. Answers may vary. Sample: elimination; neither equation is easily solved for a variable.

6. You would need more of the 15% brand, since 25% is closer to 15% than 40%.

Practice and Problem-Solving Exercises

7. **Business** A bicycle store costs $2400 per month to operate. The store pays an average of $60 per bike. The average selling price of each bicycle is $120. How many bicycles must the store sell each month to break even?

7. 40 bicycles

8. **Theater** Producing a musical costs $88,000 plus $5900 per performance. One sold-out performance earns $7500 in revenue. If every performance sells out, how many performances are needed to break even?

8. 55 performances

9. Investment You split $1500 between two savings accounts. Account A pays annual 5% interest and Account B pays 4% annual interest. After one year, you have earned a total of $69.50 in interest. How much money did you invest in each account? Explain.

9. Let x represent the amount of money invested at 5% and let y represent the amount of money invested at 4%. The solution to the system is (950, 550). $950 was invested at 5% and $850 was invested at 4%.

STEM **10. Biology** A group of scientists studied the effect of a chemical on various strains of bacteria. Strain A started with 6000 cells and decreased at a constant rate of 2000 cells per hour after the chemical was applied. Strain B started with 2000 cells and decreased at a constant rate of 1000 cells per hour after the chemical was applied. When will the strains have the same number of cells? Explain.

10. The solution to the system is (4, −2000). The solution (4, −2000) is not a viable solution because it is not possible to have −2000 cells. So, the strains will never have the same number of cells.

11. Airports A traveler is walking on a moving walkway in an airport. The traveler must walk back on the walkway to get a bag he forgot. The traveler's groundspeed is 2 ft/s against the walkway and 6 ft/s with the walkway. What is the traveler's speed off the walkway? What is the speed of the moving walkway?

11. 4 ft/s; 2 ft/s

12. Kayaking A kayaker paddles upstream from camp to photograph a waterfall and returns. The kayaker's speed while traveling upstream and downstream is shown below. What is the kayaker's speed in still water? What is the speed of the current?

12. 5.5 mi/h; 1.5 mi/h

13. Money You have a jar of pennies and quarters. You want to choose 15 coins that are worth exactly $4.35.

a. Write and solve a system of equations that models the situation.

b. Is your solution reasonable in terms of the original problem? Explain.

13. a. Let x = the number of pennies and let y = the number of quarters.

$x + y = 15$

$0.01x + 0.25y = 4.35$

The solution is 17.5 quarters and −2.5 pennies.

b. No; you can't have a negative number of coins.

Solve each system. Explain why you chose the method you used.

14. $4x + 5y = 3$
$3x - 2y = 8$

14. (2, −1); elimination method because neither equation easily solves for a variable

---

15. $2x + 7y = -20$
$y = 3x + 7$

15. (−3, −2); substitution because the second equation is already solved for y

---

16. $5x + 2y = 17$
$x - 2y = 8$

16. $\left(\frac{25}{6}, -\frac{23}{12}\right)$; Explanations may vary. Sample: substitution because one of the equations is easily solved for x

17. Reasoning Find A and B so that the system below has the solution $(2, 3)$.

$$Ax - 2By = 6$$
$$3Ax - By = -12$$

17. $A = -3$ and $B = -2$.

18. Think About a Plan A tugboat can pull a boat 24 mi downstream in 2 h. Going upstream, the tugboat can pull the same boat 16 mi in 2 h. What is the speed of the tugboat in still water? What is the speed of the current?

- How can you use the formula $d = rt$ to help you solve the problem?
- How are the tugboat's speeds when traveling upstream and downstream related to its speed in still water and the speed of the current?

18. 10 mi/h; 2 mi/h

Open-Ended **Without solving, decide which method you would use to solve each system: *graphing*, *substitution*, or *elimination*. Explain.**

19–21. Answers may vary. Samples are given.

19. $y = 3x - 1$
$y = 4x$

19. Substitution; both equations are already solved for *y*, so you can set them equal.

---

20. $3m - 4n = 1$
$3m - 2n = -1$

20. Elimination; you can just subtract the two equations, as the coefficients of *x* are the same.

---

21. $4s - 3t = 8$
$t = -2s - 1$

21. Substitution; the second equation is already solved for *y*.

22. Business A perfume maker has stocks of two perfumes on hand. Perfume A sells for \$15 per ounce. Perfume B sells for \$35 per ounce. How much of each should be combined to make a 3-oz bottle of perfume that can be sold for \$63?

22. 2.1 oz of Perfume A, 0.9 oz of Perfume B

STEM **23. Chemistry** In a chemistry lab, you have two vinegars. One is 5% acetic acid, and one is 6.5% acetic acid. You want to make 200 mL of a vinegar with 6% acetic acid How many milliliters of each vinegar do you need to mix together?

23. $66\frac{2}{3}$ mL of the 5% mixture; $133\frac{1}{3}$ mL of the 6.5% mixture

24. Boating A boat is traveling in a river with a current that has a speed of 1.5 km/h. In one hour, the boat can travel twice the distance downstream that it can travel upstream. What is the boat's speed in still water?

24. 4.5 km/h

$y - 3x = 4$
$y - 6x = 12$

25. Reasoning A student claims that the best way to solve the system at the right is by substitution. Do you agree? Explain.

25. It can also be solved by the elimination method because the variables are lined up and the coefficients of the *y*-terms are the same. So one would simply have to subtract the second equation.

26. Entertainment A contestant on a quiz show gets 150 points for every correct answer and loses 250 points for each incorrect answer. After answering 20 questions, the contestant has 200 points. How many questions has the contestant answered correctly? Incorrectly?

26. 13 correctly and 7 incorrectly

Challenge

27. Number Theory You can represent the value of any two-digit number with the expression $10a + b$, where a is the tens' place digit and b is the ones' place digit. For example, if a is 5 and b is 7, then the value of the number is $10(5) + 7$, or 57. What two-digit number is described below?

- The ones' place digit is one more than twice the tens' place digit.
- The value of the number is two more than five times the ones' place digit.

27. 37

Standardized Test Prep

28. **Mixed Nuts** You want to sell 1-lb jars of mixed peanuts and cashews for $5. You pay $3 per pound for peanuts and $6 per pound for cashews. You plan to combine 4 parts peanuts and 1 part cashews to make your mix. You have spent $70 on materials to get started. How many jars must you sell to break even?

28. 50 jars

29. Last year, one fourth of the students in your class played an instrument. This year, 6 students joined the class. Four of the new students play an instrument. Now, one third of the students play an instrument. How many students are in your class now?

(A) 18 (B) 24 (C) 30 (D) 48

29. C

Standardized Test Prep

30. Which answer choice shows $2x - y = z$ correctly solved for y?

F $y = 2x + z$ G $y = 2x - z$ H $y = -2x + z$ I $y = -2x - z$

30. G

31. What is an equation of a line passing through the points $(3, 1)$ and $(4, 3)$ written in slope-intercept form?

The slope of the line is $\frac{3 - 1}{4 - 3} = 2$.

So $y - 1 = 2(x - 3)$, or $y - 1 = 2x - 6$. The equation of the line passing through the points (3, 1) and (4, 3) is $y = 2x - 5$.

Mixed Review

Solve each system using elimination.

32. $x + 3y = 11$
$2x + 3y = 4$

32. $(-7, 6)$

33. $2x + 4y = -12$
$-6x + 5y = 2$

33. $(-2, -2)$

34. $5x + 8y = 40$
$3x - 10y = -13$

34. $(4, 2.5)$

Get Ready! **To prepare for Lesson 6-5, do Exercises 35–37.**

35. $3a + 5 > 20$

36. $2d - 3 \geq 4d + 2$

37. $3(q + 4) \leq -2q - 8$

35. $a > 5$

36. $d \leq -2.5$

37. $q \leq -4$

End 6-4