Objectives To solve systems of linear inequalities by graphing
To model real-world situations using systems of linear inequalities

PURPOSE To model a real-world situation with a system of linear inequalities in two variables

ESSENTIAL QUESTION How can you solve a system of equations or inequalities?

Practice and Problem-Solving Exercises

Determine whether the ordered pair is a solution of the given system.

7. $(2, 12)$;
$y > 2x + 4$
$y < 3x + 7$

7. yes

8. $(8, 2)$;
$3x - 2y \leq 17$
$0.3x + 4y > 9$

8. no

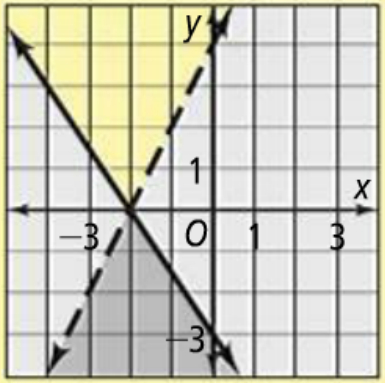
9. $(-3, 17)$;
$y > -5x + 2$
$y \geq -3x + 7$

9. no

Solve each system of inequalities by graphing.

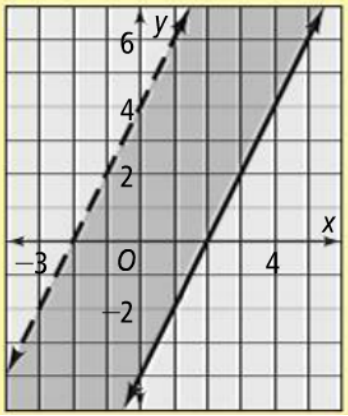
10. $y < 2x + 4$
$-3x - 2y \geq 6$

10.



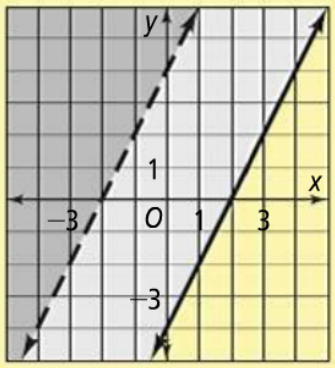
11. $y < 2x + 4$
$2x - y \leq 4$

11.



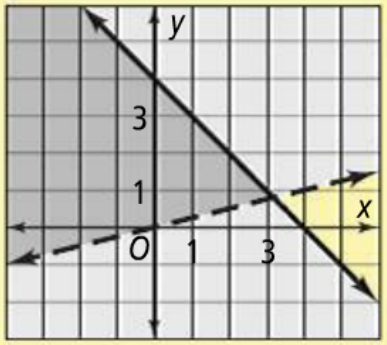
12. $y > 2x + 4$
$2x - y \leq 4$

12.



13. $y > \frac{1}{4}x$
$y \leq -x + 4$

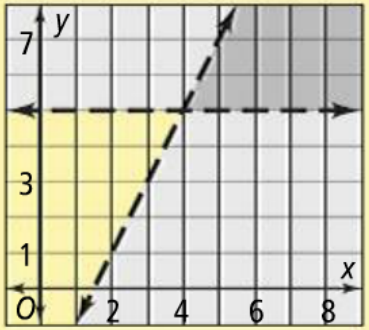
13.



Solve each system of inequalities by graphing.

14. $y < 2x - 3$
$y > 5$

14.



15. $y \leq -\frac{1}{3}x + 7$
$y \geq -x + 1$

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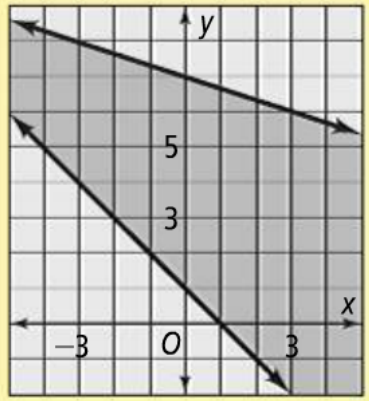

16. $x + 2y \leq 10$
$x + 2y \geq 9$

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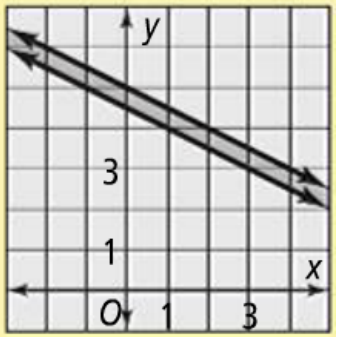

17. $y \geq -x + 5$
$y \leq 3x - 4$

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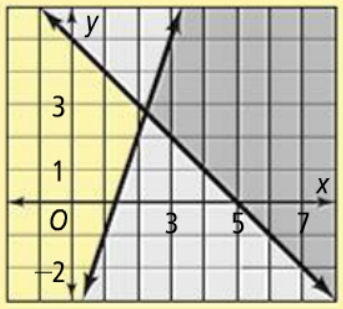
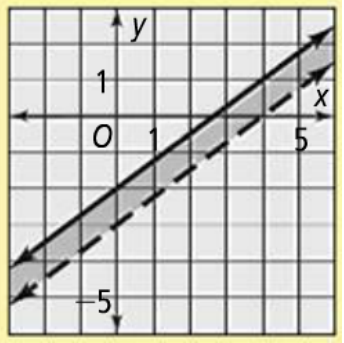


Solve each system of inequalities by graphing.

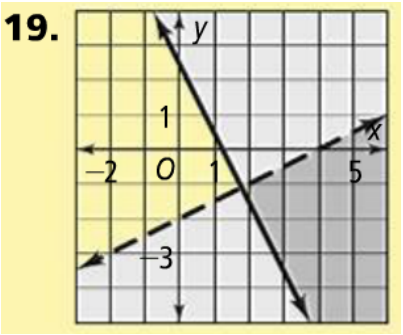
18. $y \leq 0.75x - 2$
$y > 0.75x - 3$

18.



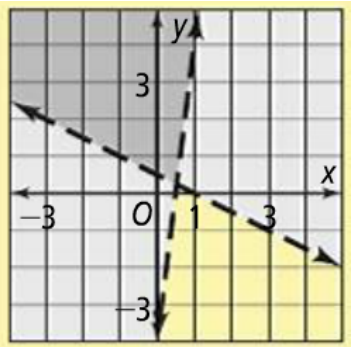
19. $8x + 4y \geq 10$
$3x - 6y > 12$

19.



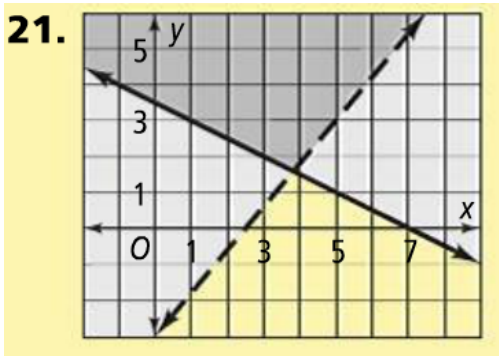
20. $2x - \frac{1}{4}y < 1$
$4x + 8y > 4$

20.



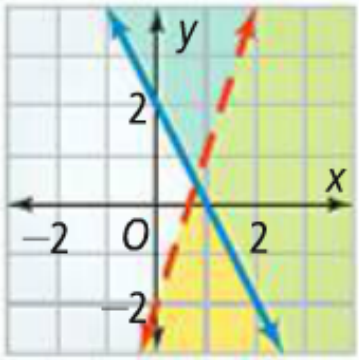
21. $6x - 5y < 15$
$x + 2y \geq 7$

21.



Write a system of inequalities for each graph.

22.



22. $y > 3x - 2,\ y \geq -2x + 2$

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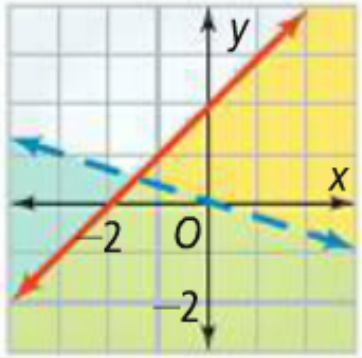
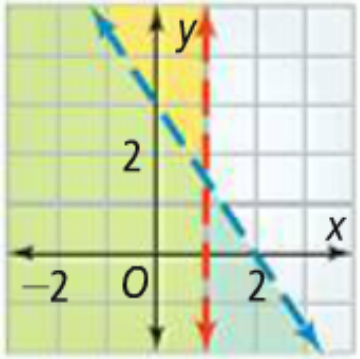


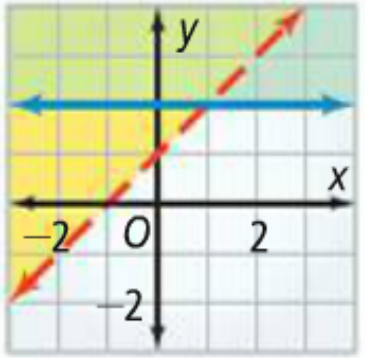
23. $y \leq x + 2,\ y < -\frac{1}{3}x$

24.



24. $x < 1,\ y < -\frac{3}{2}x + 3$

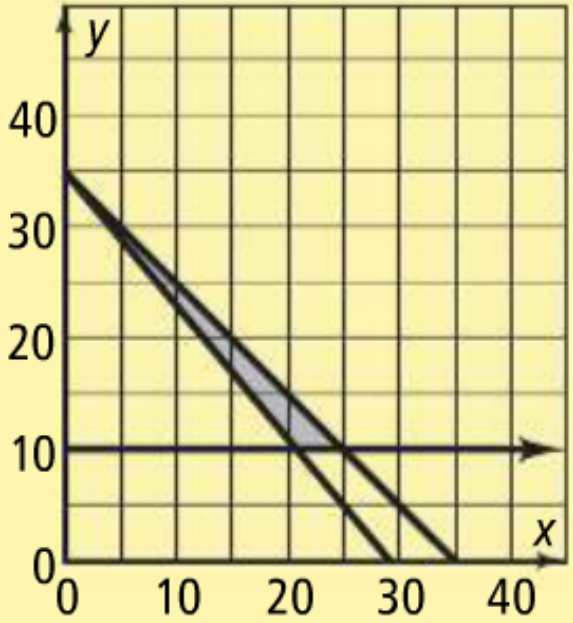
25.



25. $y \geq 2,\ y > x + 1$

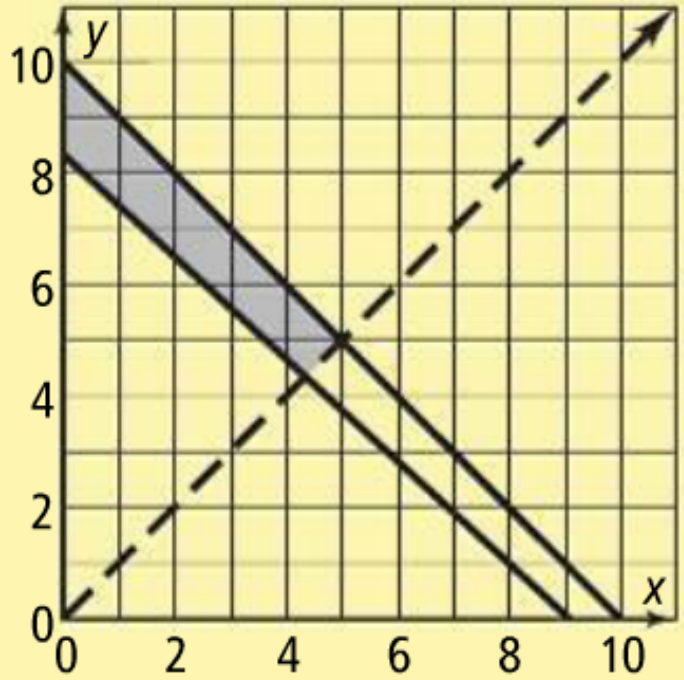
26. Earnings Suppose you have a job mowing lawns that pays $12 per hour. You also have a job at a clothing store that pays $10 per hour. You need to earn at least $350 per week, but you can work no more than 35 h per week. You must work a minimum of 10 h per week at the clothing store. What is a graph showing how many hours per week you can work at each job?

26. Let x = hours worked at mowing lawns, let y = hours worked at clothing store.



27. Driving Two friends agree to split the driving on a road trip from Philadelphia, Pennsylvania, to Denver, Colorado. One friend drives at an average speed of 60 mi/h. The other friend drives at an average speed of 55 mi/h. They want to drive at least 500 mi per day. They plan to spend no more than 10 h driving each day. The friend who drives slower wants to drive fewer hours. What is a graph showing how they can split the driving each day?

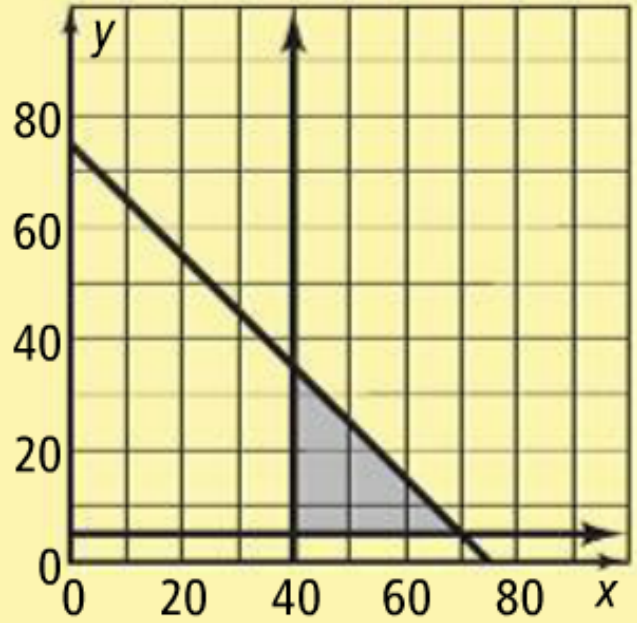
27. Let x = hours driven by slower driver, let y = hours driven by faster driver.



28. Think About a Plan You are fencing in a rectangular area for a garden. You have only 150 ft of fence. You want the length of the garden to be at least 40 ft. You want the width of the garden to be at least 5 ft. What is a graph showing the possible dimensions your garden could have?

- What variables will you use? What will they represent?
- How many inequalities do you need to write?

28. Let x = length of garden in feet, let y = width of garden in feet.



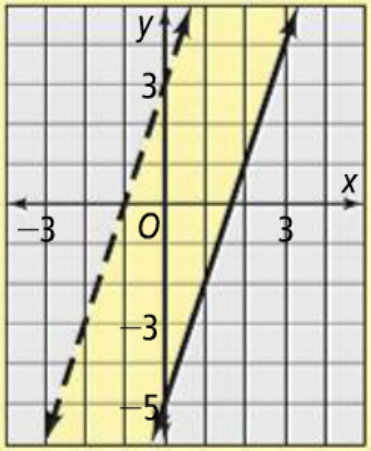
29. **a.** Graph the system $y > 3x + 3$ and $y \leq 3x - 5$.

b. **Writing** Will the boundary lines $y = 3x + 3$ and $y = 3x - 5$ ever intersect? How do you know?

c. Do the shaded regions in the graph from part (a) overlap?

d. Does the system of inequalities have any solutions? Explain.

29. a.

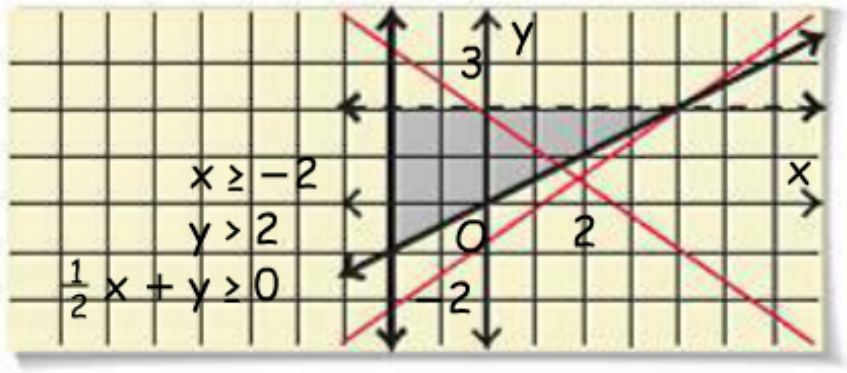


b. No; they have the same slope and different *y*-intercepts, so they will never intersect.

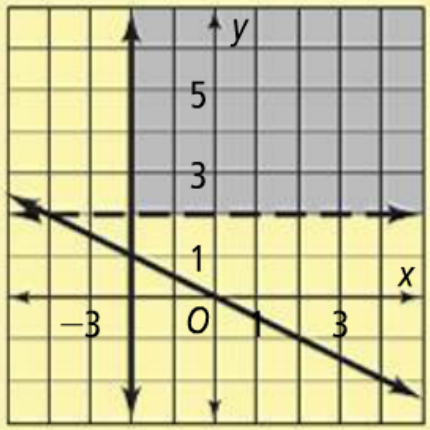
c. no

d. No; there are no points that satisfy both inequalities.

30. Error Analysis A student graphs the system as shown below. Describe and correct the student's error.



30. The student graphed $y \geq \frac{1}{2}x$, but he should have graphed $y \geq -\frac{1}{2}x$, and he shaded below $y = 2$, but he should have shaded above.



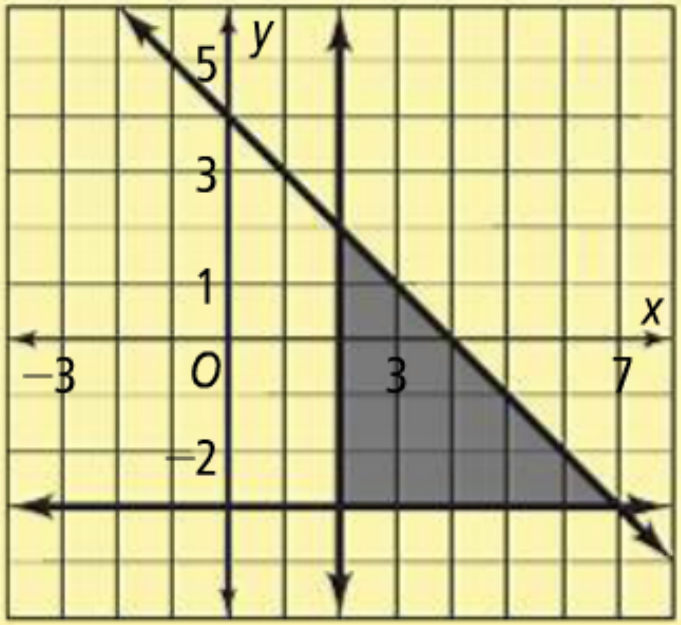
31. Gift Certificates You received a $100 gift certificate to a clothing store. The store sells T-shirts for $15 and dress shirts for $22. You want to spend no more than the amount of the gift certificate. You want to leave at most $10 of the gift certificate unspent. You need at least one dress shirt. What are all of the possible combinations of T-shirts and dress shirts you could buy?

31. You can buy 5 T-shirts and 1 dress shirt or 2 T-shirts and 3 dress shirts.

32. a. Geometry Graph the system of linear inequalities.
b. Describe the shape of the solution region.
c. Find the vertices of the solution region.
d. Find the area of the solution region.

$x \geq 2$
$y \geq -3$
$x + y \leq 4$

32. a.



b. right triangle

c. (2, 2), (2, −3), (7, −3)

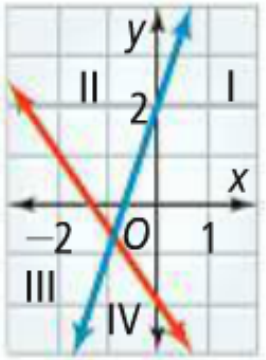
d. 12.5 units^2

33. Which region represents the solution of the system?

$$y \leq -\frac{3}{2}x - 2$$
$$3y - 9x \geq 6$$

(A) I (C) III

(B) II (D) IV



33. C

Open-Ended **Write a system of linear inequalities with the given characteristic.**

34. All solutions are in Quadrant III.

35. There are no solutions.

34 and 35 answers will vary between students. Problem 35 has two inequalities with no points in common.

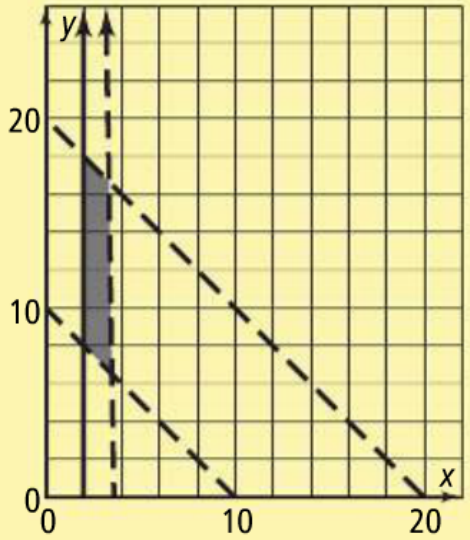
36. Business A jeweler plans to produce a ring made of silver and gold. The price of gold is about \$25 per gram. The price of silver is approximately \$.40 per gram. She considers the following in deciding how much gold and silver to use in the ring.

- The total mass must be more than 10 g but less than 20 g.
- The ring must contain at least 2 g of gold.
- The total cost of the gold and silver must be less than \$90.

a. Write and graph the inequalities that describe this situation.

b. For one solution, find the mass of the ring and the cost of the gold and silver.

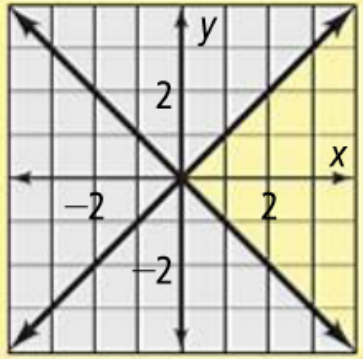
36. a. Let x = g of gold and let y = g of silver; $25x + 0.40y < 90$, $10 < x + y$, $x + y < 20$; $x \geq 2$.



b. Answers may vary. Sample: One solution is (3, 11), which represents 3 g of gold and 11 g of silver. $10 < 3 + 11 < 20$ and $3 > 2$. The cost of this ring will be $3(25) + 11(0.40) = 79.4$, or \$79.40.

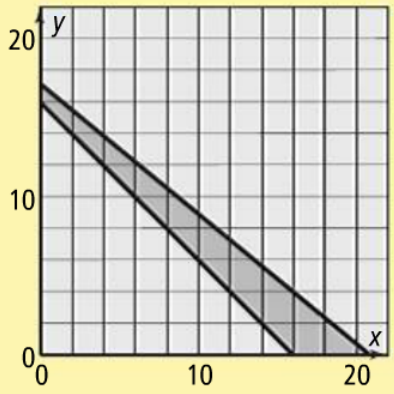
37. Solve $|y| \geq x$. (*Hint:* Write two inequalities and then graph them.)

37.



38. Student Art A teacher wants to post a row of student artwork on a wall that is 20 ft long. Some pieces are 8.5 in. wide. Other pieces are 11 in. wide. She is going to leave 3 in. of space to the left of each art piece. She wants to post at least 16 pieces of art. Write and graph a system of inequalities that describes how many pieces of each size she can post.

38. $11.5x + 14y \leq 240$
$x + y \geq 16$;



39. The point $(-3, 11)$ is a solution of which of the following systems?

(A) $y \geq x - 2$, $2x + y \leq 5$ (B) $y > x + 8$, $3x + y > 2$ (C) $y > -x + 8$, $2x + 3y \geq 7$ (D) $y \leq -3x + 1$, $x - y \geq -15$

39. A

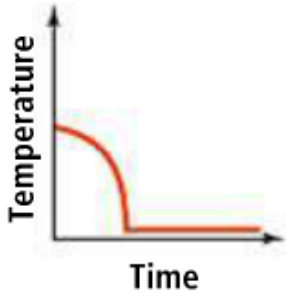
40. A plane has 18 passengers. Some have 1 bag and others have 2 bags. There are a total of 27 bags. Let b = the number of passengers with 1 bag and t = the number of passengers with 2 bags. Which system describes this situation?

(F) $b + t = 27$, $b + 2t = 18$ (G) $t = 18 - b$, $b + 2t = 27$ (H) $b + t = 18$, $b = 27 + 2t$ (I) $b = 18 - t$, $b + 2t = 18$

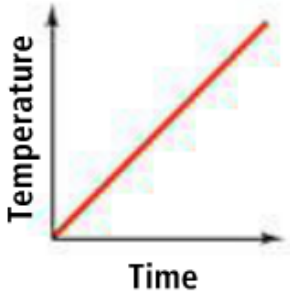
40. G

41. You fill your glass with ice and then add room-temperature water. Which graph best represents the change in temperature of the glass?

A



B



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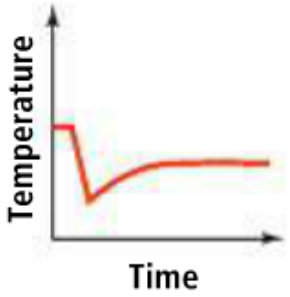
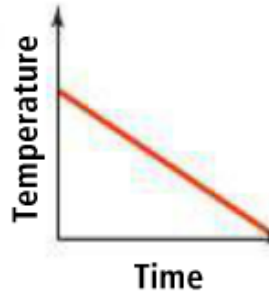


D



41. C

42. Suppose the line through points $(-1, 6)$ and $(x, 2)$ is perpendicular to the graph of $2x + y = 3$.

a. Find the value of x. Show your work.

b. What is an equation of the line perpendicular to the graph of $2x + y = 3$ and through the given points?

c. What ordered pair is a solution of both equations? Explain.

42. a. [4] $2x + y = 3$, so $y = -2x + 3$ and the slope of the line is -2. Therefore, the slope of the line containing the points must be equal to $\frac{1}{2}$.

$\frac{2 - 6}{x - (-1)} = \frac{1}{2}$ or $\frac{-4}{x + 1} = \frac{1}{2}$

$(-4)(2) = 1(x + 1)$

$-8 = x + 1$

$-9 = x$

b. $m = \frac{1}{2}$ and contains the points $(-1, 6)$ and $(-9, 2)$. Use the slope and either one of the points.

$y - y_1 = m(x - x_1)$

$y - 6 = \frac{1}{2}(x - (-1))$

$y - 6 = \frac{1}{2}(x + 1)$

$y - 6 = \frac{1}{2}x + \frac{1}{2}$

$y = \frac{1}{2}x + \frac{13}{2}$ or $x - 2y = -13$

c. Solve the system

$2x + y = 3$ $\quad 2(2x + y = 3)$

$4x + 2y = 6$

$x - 2y = -13$ $\quad x - 2y = -13$

$x - 2y = -13$

$5x = -7$ or $x = -\frac{7}{5}$

Now substitute $x = -\frac{7}{5}$ into either equation to solve for y. $2\left(\frac{-7}{5}\right) + y = 3$, so $y = 3 + \frac{14}{5}$ or $y = \frac{29}{5}$.

So the point $\left(-\frac{7}{5}, \frac{29}{5}\right)$ is a solution of both equations.

Mixed Review

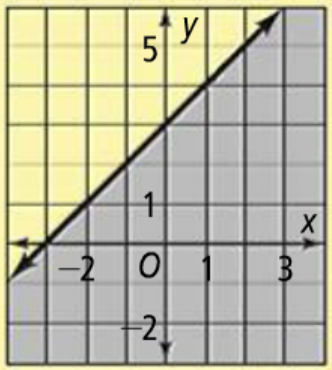
Graph each linear inequality.

43. $y - x \leq 3$

44. $3y + x > 4$

45. $y \leq 5$

43.



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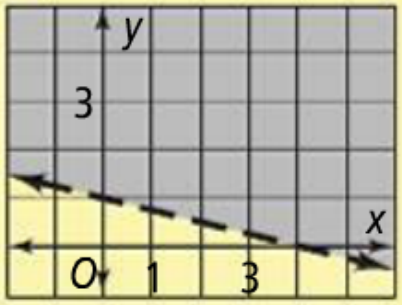

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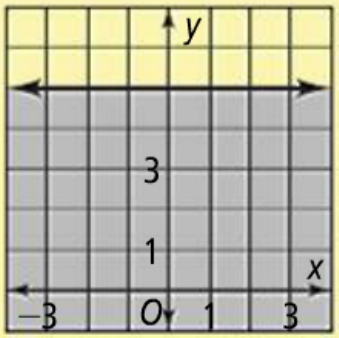

Get Ready! **To prepare for Lesson 7-1, do Exercises 46–48.**

Simplify each expression.

46. $(1 + 3)^2 - (1 + 3)$

47. $4^3 + 5^2 + (4 - 3)^1$

48. $7^2 + 2(3^3 + 5)$

46. 12

47. 90

48. 113

Notes 6-6