Objectives To graph linear inequalities in two variables
To use linear inequalities when modeling real-world situations

Lesson Check

Do you know HOW?

1. Is $(-1, 4)$ a solution of the inequality $y < 2x + 5$?

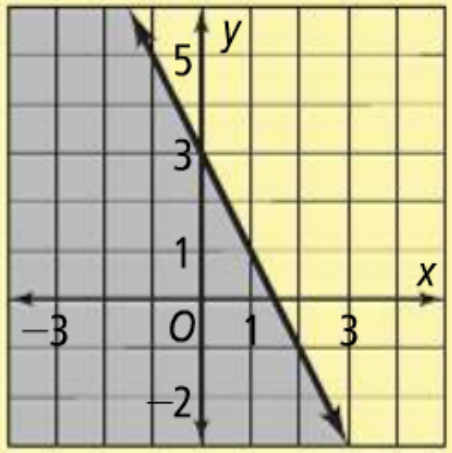
1. no

Graph each linear inequality.

2. $y \leq -2x + 3$

3. $x < -1$

2.



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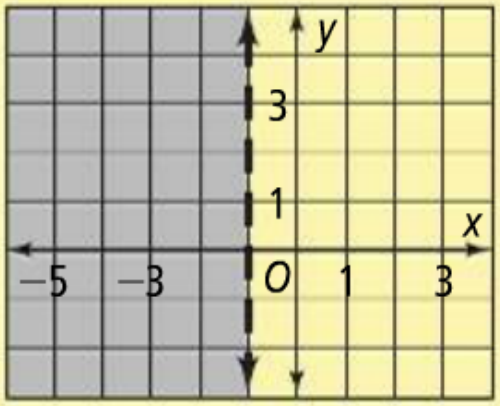
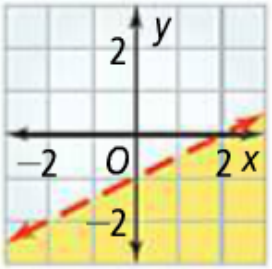


4. What is an inequality that represents the graph at the right?



4. $y < \frac{1}{2}x - 1$

Do you UNDERSTAND? MATHEMATICAL PRACTICES

5. Vocabulary How is a linear inequality in two variables like a linear equation in two variables? How are they different?

5. Answers will vary. Sample: The solutions of a linear equation and a linear inequality are coordinates of the points that make the equation or inequality true. The graph of a linear equation is a line, but the graph of a linear inequality is a region of the coordinate plane.

6. Writing To graph the inequality $y < \frac{3}{2}x + 3$, do you shade above or below the boundary line? Explain.

6. Since the inequality is already solved for y, the $<$ symbol means you should shade below the boundary line. All of these shaded points will make the inequality true.

7. Reasoning Write an inequality that describes the region of the coordinate plane *not* included in the graph of $y < 5x + 1$.

7. $y \geq 5x + 1$

Practice and Problem-Solving Exercises

Determine whether the ordered pair is a solution of the linear inequality.

8. $y \leq -2x + 1; (2, 2)$

9. $x < 2; (-1, 0)$

10. $y \geq 3x - 2; (0, 0)$

8. not a solution

9. solution

10. solution

Determine whether the ordered pair is a solution of the linear inequality.

11. $y > x - 1; (0, 1)$

11. solution

12. $y \geq -\frac{2}{5}x + 4; (0, 0)$

12. not a solution

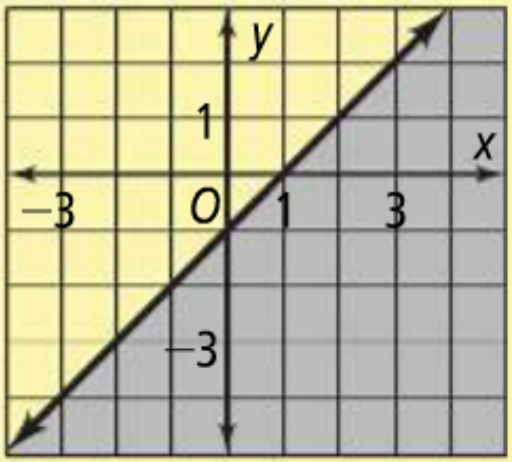
13. $3y > 5x - 12; (-6, 1)$

13. solution

Graph each linear inequality.

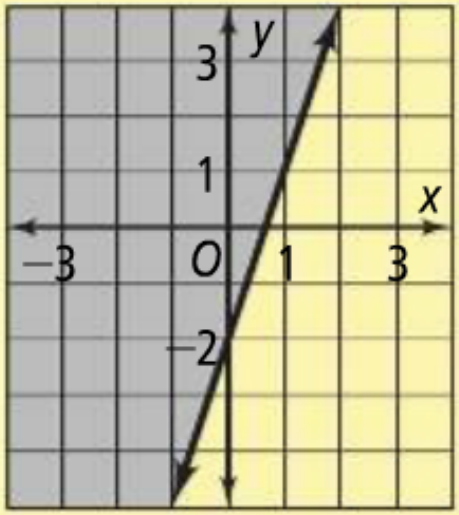
14. $y \leq x - 1$

14.



15. $y \geq 3x - 2$

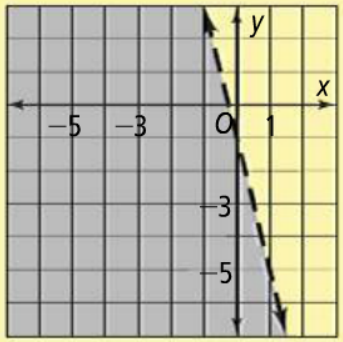
15.



Graph each linear inequality.

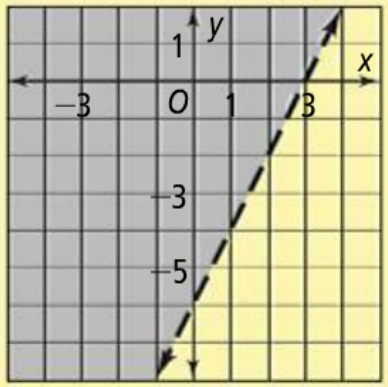
16. $y < -4x - 1$

16.



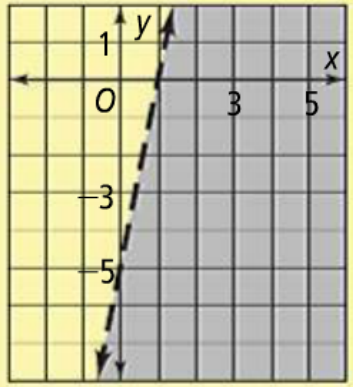
17. $y > 2x - 6$

17.



18. $y < 5x - 5$

18.



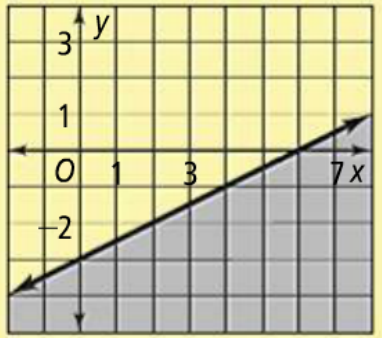
Graph each linear inequality.

19. $y \leq \frac{1}{2}x - 3$

20. $y > -3x$

21. $y \geq -x$

19.



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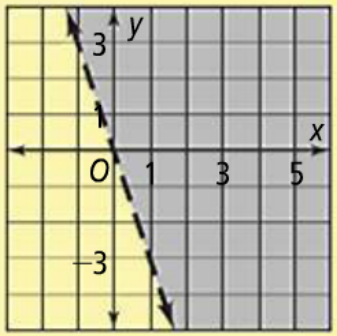

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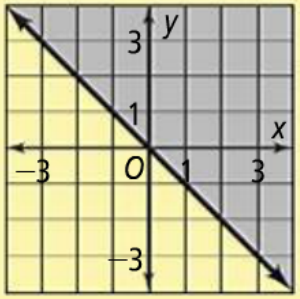
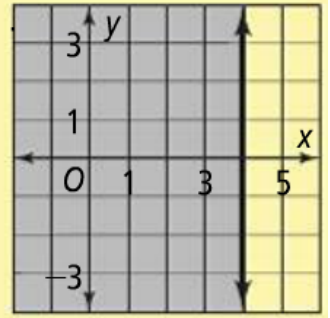


Graph each inequality in the coordinate plane.

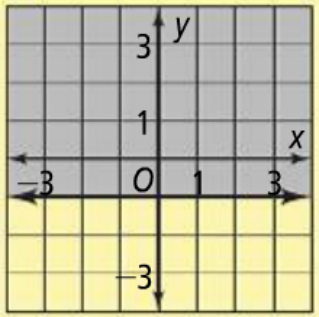
22. $x \leq 4$

22.



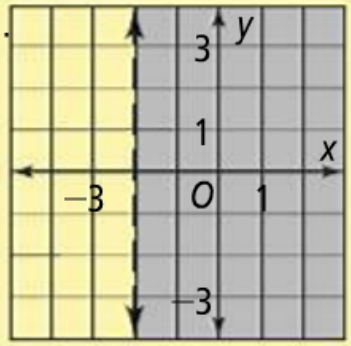
23. $y \geq -1$

23.



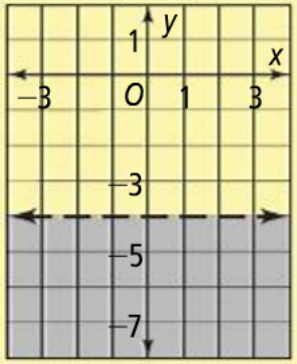
24. $x > -2$

24.



25. $y < -4$

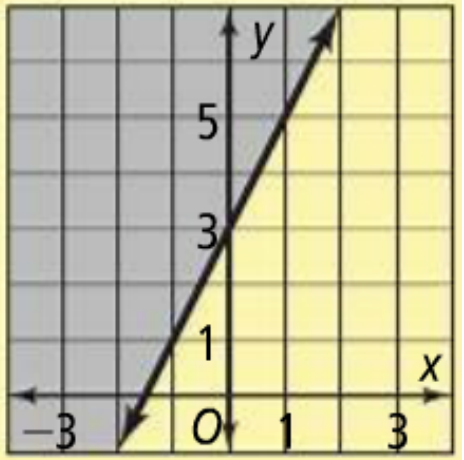
25.



Graph each inequality in the coordinate plane.

26. $-2x + y \geq 3$

26.



27. $x + 3y < 15$

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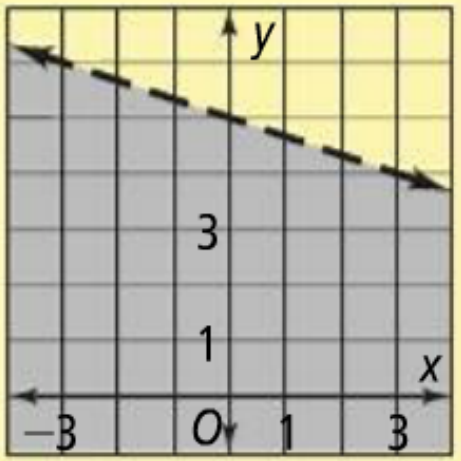
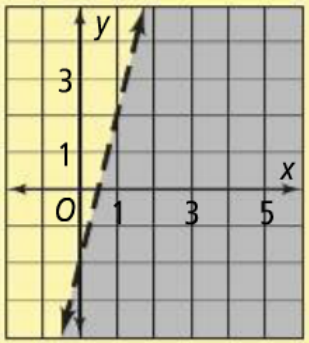


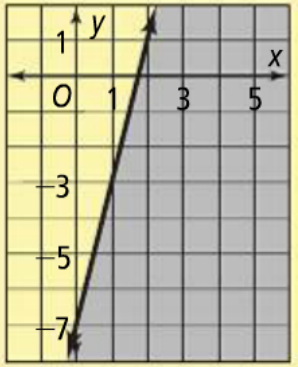
28. $4x - y > 2$

28.



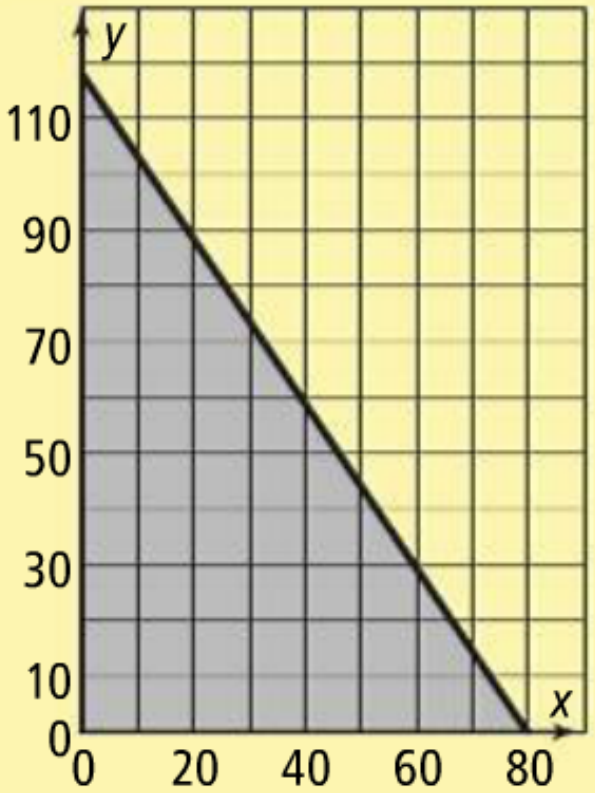
29. $-x + 0.25y \leq -1.75$

29.



30. Carpentry You budget \$200 for wooden planks for outdoor furniture. Cedar costs \$2.50 per foot and pine costs \$1.75 per foot. Let x = the number of feet of cedar and let y = the number of feet of pine. What is an inequality that shows how much of each type of wood can be bought? Graph the inequality. What are three possible amounts of each type of wood that can be bought within your budget?

30.

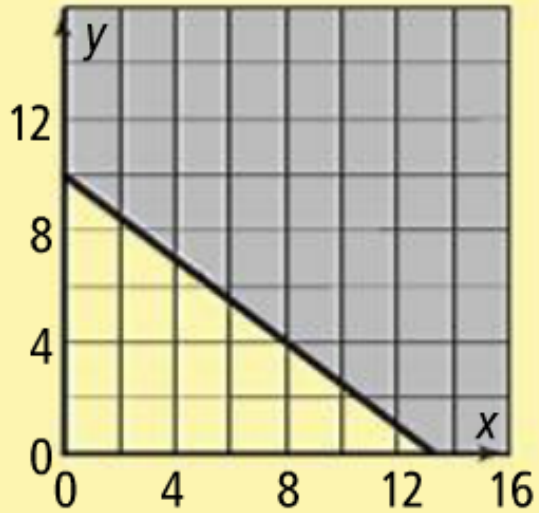


30. $2.5x + 1.75y \leq 200$

Answers may vary. Sample: 10 ft of cedar and 80 ft of pine; 20 ft of cedar and 50 ft of pine; 60 ft of cedar and 20 ft of pine.

31. Business A fish market charges \$9 per pound for cod and \$12 per pound for flounder. Let x = the number of pounds of cod. Let y = the number of pounds of flounder. What is an inequality that shows how much of each type of fish the store must sell today to reach a daily quota of at least \$120? Graph the inequality. What are three possible amounts of each fish that would satisfy the quota?

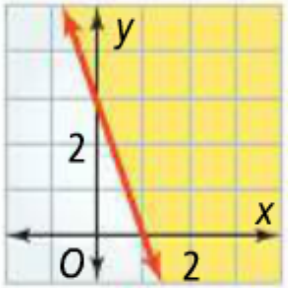
31. $9x + 12y \geq 120$



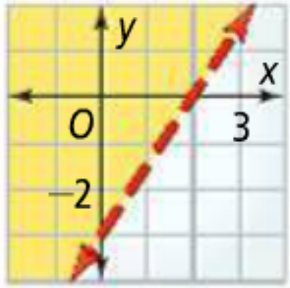
Answers may vary. Sample: 4 lb of cod and 12 lb of flounder; 10 lb of cod and 10 lb of flounder; 12 lb of cod and 4 lb of flounder

Write a linear inequality that represents each graph.

32.



33.



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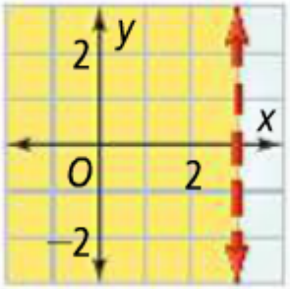


32. $y \geq -3x + 3$

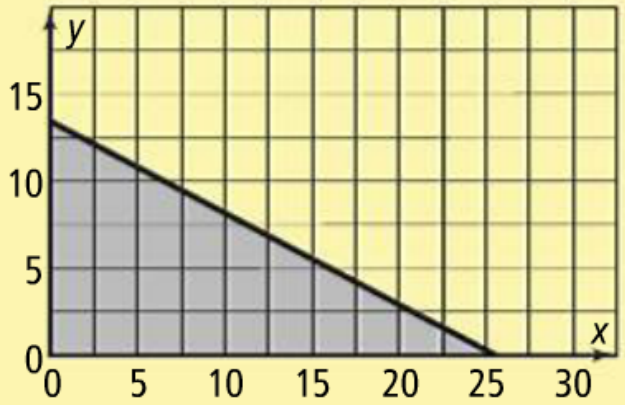
33. $y > \frac{3}{2}x - 3$

34. $x < 3$

35. Think About a Plan A truck that can carry no more than 6400 lb is being used to transport refrigerators and upright pianos. Each refrigerator weighs 250 lb and each piano weighs 475 lb. Write and graph an inequality to show how many refrigerators and how many pianos the truck could carry. Will 12 refrigerators and 8 pianos overload the truck? Explain.

- What inequality symbol should you use?
- Which side of the boundary line should you shade?

35. $250x + 475y \leq 6400$, where x represents the number of refrigerators and y represents the number of pianos



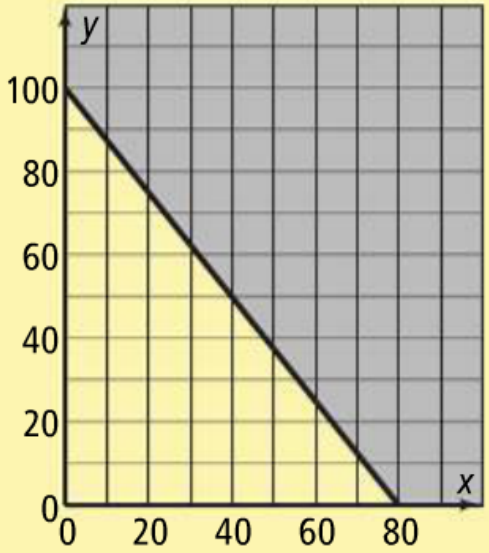
Yes; the point (12, 8) is not in the shaded region.

36. Employment A student with two summer jobs earns \$10 per hour at a cafe and \$8 per hour at a market. The student would like to earn at least \$800 per month.

a. Write and graph an inequality to represent the situation.

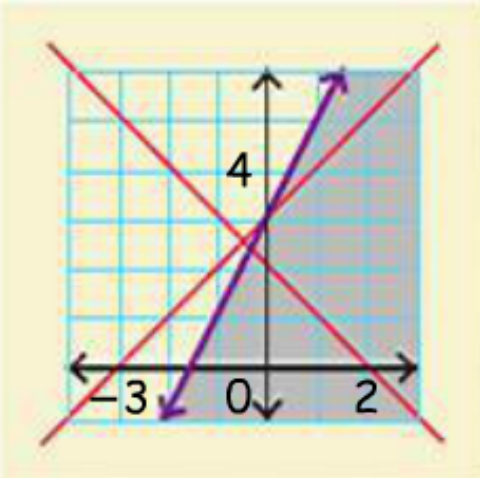
b. The student works at the market for 60 h per month and can work at most 90 h per month. Can the student earn at least \$800 each month? Explain how you can you use your graph to determine this.

36. a. Let x = hours at the cafe and let y = hours at the market;
$10x + 8y \geq 800$

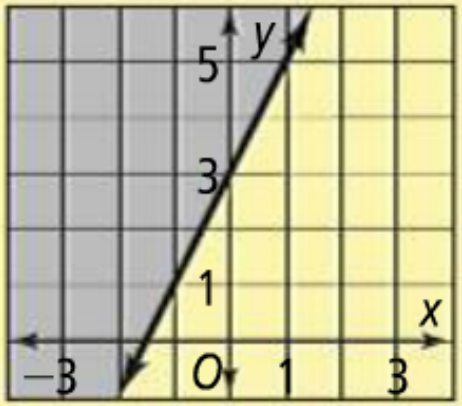


b. No; the point (30, 60) does not lie in the shaded region of the graph.

37. Error Analysis A student graphed $y \geq 2x + 3$ as shown at the right. Describe and correct the student's error.



37. The student graphed $y \leq 2x + 3$ instead of $y \geq 2x + 3$. The other side of the line should be shaded.



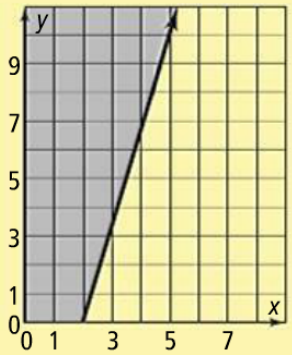
38. Writing When graphing an inequality, can you always use (0, 0) as a test point to determine where to shade? If not, how would you choose a test point?

38. You could not use the point (0, 0) in the case that (0, 0) lies on the boundary line. If that were the case, you would have to choose any other point that was not on the boundary line.

Challenge

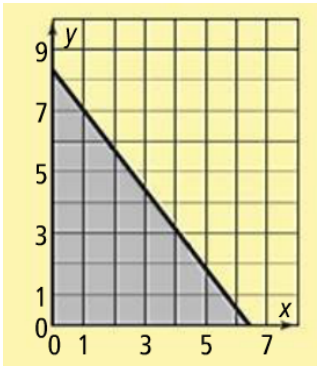
39. Music Store A music store sells used CDs for $5 each and buys used CDs for $1.50 each. You go to the store with $20 and some CDs to sell. You want to have at least $10 left when you leave the store. Write and graph an inequality to show how many CDs you could buy and sell.

39. $-5x + 1.5y \geq -10$, where x is the number of CDs bought and y is the number sold; actual solutions include only points representing whole numbers of CDs bought and sold.



40. Groceries At your grocery store, milk normally costs \$3.60 per gallon. Ground beef costs \$3 per pound. Today there are specials: Milk is discounted \$.50 per gallon, and ground beef is 20% off. You want to spend no more than \$20. Write and graph a linear inequality to show how many gallons of milk and how many pounds of ground beef you can buy today.

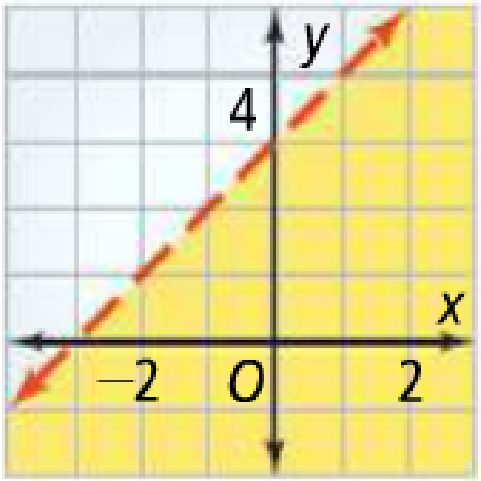
40. $3.10x + 2.40y \leq 20$, where x represents the number of gallons of milk and y represents the number of pounds of hamburger



41. Reasoning You are graphing a linear inequality of the form $y > mx + b$. The point (1, 2) is not a solution, but (3, 2) is. Is the slope of the boundary line *positive, negative, zero,* or *undefined*? Explain.

41. The slope must be negative; in order for the point (3, 2) to be above the line and the point (1, 2) to be below the line, the boundary line must be sloping downward. If the line had a positive slope, sloping upward, then the point (1, 2) would be above the line and would satisfy $y > mx + b$, which is not what is given.

Standardized Test Prep



Look at the intercepts of the line:

(-3, 0) and (0, 3)

When x = 0 y = 3

when y = 0 x = -3

find the equation of the line, then look at whether it should be

> or <

42. What is the equation of the graph shown?

(A) $y + x \geq -3$

(B) $y - x \geq 3$

(C) $x - y > -3$

(D) $y > -x + 3$

42. C

Standardized Test Prep

43. You secure pictures to your scrapbook using 3 stickers. You started with 24 stickers. There are now 2 pictures in your scrapbook. You write the equation $3(x + 2) = 24$ to find the number x of additional pictures you can put in your scrapbook. How many more pictures can you add?

(F) 4
(G) 6
(H) 8
(I) 12

43. G

44. At Market A, 1-lb packages of rice are sold for the price shown. At Market B, rice is sold in bulk for the price shown. For each market, write a function describing the cost of buying rice in terms of the weight. How are the domains of the two functions different?

$2.00 $2.00/lb

44. The cost at Market A is given by $C(x) = 2x$, where x is a nonnegative integer. The cost at Market B is given by $C(x) = 2x$, where x is a nonnegative real number. In Market A you can only buy multiples of 1-lb packages; at Market B you can buy any amount.

Mixed Review

45. **Small Business** An electrician spends $12,000 on initial costs to start a new business. He estimates his expenses at $25 per day. He expects to earn $150 per day. If his estimates are correct, after how many working days will he break even?

45. 96 days

46. What compound inequality represents the phrase "all real numbers that are greater than 2 and less than or equal to 7"? Graph the solutions.

46. $2 < x \leq 7$

Get Ready! **To prepare for Lesson 6-6, do Exercises 47–49.**

Solve each system by graphing. Tell whether the system has *one solution*, *infinitely many solutions*, or *no solution*.

47. $y = \frac{3}{2}x$
$-2x + y = 3$

48. $3x + y = 6$
$2x - y = 4$

49. $x + y = 11$
$x + y = 16$

47. one solution:

(−6, −9)

48. one solution:

(2, 0)

49. no solution

Notes 6-5