

6-3

Solving Systems Using Elimination

Objective To solve systems by adding or subtracting to eliminate a variable

PURPOSE To use the properties of equality to solve a real-world problem that can be represented by a system of linear equations

A cafeteria sells fresh fruit by weight. All apples weigh the same, and all oranges weigh the same. What is the weight of an apple? What is the weight of an orange? How do you know?



If we take away 3 oranges and two apples from the second scale, we take away 37 oz, leaving 2 oranges weighing 14 oz, so each orange is 7 oz.

If an orange is 7oz
the two apples on the left
weigh 16 oz (8 each)

If an orange is 7oz
the two apples on the right
weigh 16 oz (8 each)

Practice and Problem-Solving Exercises

Solve each system using elimination.

$$\begin{aligned} 7. \quad & 3x + 3y = 27 \\ & x - 3y = -11 \end{aligned}$$

$$7. (4, 5)$$

$$\begin{aligned} 8. \quad & -x + 5y = 13 \\ & x - y = 15 \end{aligned}$$

$$8. (22, 7)$$

$$\begin{aligned} 9. \quad & 2x + 4y = 22 \\ & 2x - 2y = -8 \end{aligned}$$

$$9. (1, 5)$$

Solve each system using elimination.

10. $4x - 7y = 3$
 $x - 7y = -15$

10. $(6, 3)$

11. $5x - y = 0$
 $3x + y = 24$

11. $(3, 15)$

12. $6x + 5y = 39$
 $3x + 5y = 27$

12. $(4, 3)$

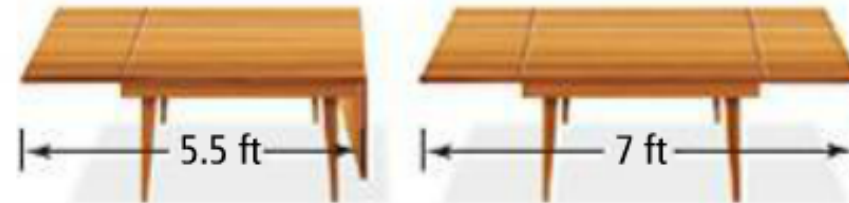
- 13. Talent Show** Your school's talent show will feature 12 solo acts and 2 ensemble acts. The show will last 90 min. The 6 solo performers judged best will give a repeat performance at a second 60-min show, which will also feature the 2 ensemble acts. Each solo act lasts x minutes, and each ensemble act lasts y minutes.
- Write a system of equations to model the situation.
 - Solve the system from part (a). How long is each solo act? How long is each ensemble act?

13. a. $12x + 2y = 90$

$6x + 2y = 60$

b. solo act: 5 min; ensemble act: 15 min

- 14. Furniture** A carpenter is designing a drop-leaf table with two drop leaves of equal size. The lengths of the table when one leaf is folded up and when both leaves are folded up are shown. How long is the table when no leaves are folded up?



14. 4 ft

Solve each system using elimination.

$$\begin{aligned} 15. \quad 2x + 3y &= 9 \\ x + 5y &= 8 \end{aligned}$$

$$15. (3, 1)$$

$$\begin{aligned} 16. \quad 3x + y &= 5 \\ 2x - 2y &= -2 \end{aligned}$$

$$16. (1, 2)$$

$$\begin{aligned} 17. \quad 6x + 4y &= 42 \\ -3x + 3y &= -6 \end{aligned}$$

$$17. (5, 3)$$

Solve each system using elimination.

$$\begin{aligned} 18. \quad 3x + 2y &= 17 \\ 2x + 5y &= 26 \end{aligned}$$

$$18. (3, 4)$$

$$\begin{aligned} 19. \quad 6x - 3y &= 15 \\ 7x + 4y &= 10 \end{aligned}$$

$$19. (2, -1)$$

$$\begin{aligned} 20. \quad 5x - 9y &= -43 \\ 3x + 8y &= 68 \end{aligned}$$

$$20. (4, 7)$$

Tell whether the system has *one solution*, *infinitely many solutions*, or *no solution*.

$$\begin{aligned} 21. \quad &9x + 8y = 15 \\ &9x + 8y = 30 \end{aligned}$$

21. no solution

$$\begin{aligned} 22. \quad &3x + 4y = 24 \\ &6x + 8y = 24 \end{aligned}$$

22. no solution

$$\begin{aligned} 23. \quad &5x - 3y = 10 \\ &10x + 6y = 20 \end{aligned}$$

23. one solution

Tell whether the system has *one solution*, *infinitely many solutions*, or *no solution*.

24. $2x - 5y = 17$
 $6x - 15y = 51$

24. infinitely many solutions

25. $4x - 7y = 15$
 $-8x + 14y = -30$

25. infinitely many solutions

26. $4x - 8y = 15$
 $-5x + 10y = -30$


26. no solution




27. Think About a Plan A photo studio offers portraits in 8×10 and wallet-sized formats. One customer bought two 8×10 portraits and four wallet-sized portraits and paid \$52. Another customer bought three 8×10 portraits and two wallet-sized portraits and paid \$50. What is the cost of an 8×10 portrait? What is the cost of a wallet-sized portrait?

- Can you eliminate a variable simply by adding or subtracting?
- If not, how many of the equations do you need to multiply by a constant?

27. \$12; \$7

-  **28. Reasoning** A toy store worker packed two boxes of identical dolls and plush toys for shipping in boxes that weigh 1 oz when empty. One box held 3 dolls and 4 plush toys. The worker marked the weight as 12 oz. The other box held 2 dolls and 3 plush toys. The worker marked the weight as 10 oz. Explain why the worker must have made a mistake.

28. The solution $(-3, 5)$ does not make sense because a plush toy can't have a negative weight.

-  **29. Error Analysis** A student solved a system of equations by elimination. Describe and correct the error made in the part of the solution shown.

$$\begin{array}{r}
 \cancel{5x + 4y = 2} \quad \text{---} \times 3 \rightarrow \cancel{15x + 12y = 6} \\
 \cancel{3x + 3y = -3} \quad \text{---} \times 4 \rightarrow \cancel{12x + 12y = -3} \\
 \hline
 \quad \quad \quad 3x + 0 = 9 \\
 \quad \quad \quad x = 3
 \end{array}$$

- 29.** The student forgot to multiply the constant in the second equation by 4.
- $$\begin{array}{l}
 15x + 12y = 6 \\
 12x + 12y = -12 \\
 \text{so,} \quad 3x = 18 \\
 \quad \quad x = 6
 \end{array}$$

- 30. Nutrition** Half a pepperoni pizza plus three fourths of a ham-and-pineapple pizza contains 765 Calories. One fourth of a pepperoni pizza plus a whole ham-and-pineapple pizza contains 745 Calories. How many Calories are in a whole pepperoni pizza? How many Calories are in a whole ham-and-pineapple pizza?

30. 660 Calories; 580 Calories



- 31. Open-Ended** Write a system of equations that can be solved efficiently by elimination. Explain what you would do to eliminate one of the variables. Then solve the system.

31. Answers may vary. Sample:

$$3x - 2y = 7$$

$$5x + 2y = 33$$

Because the coefficients of the y -terms are already opposites, simply add the two equations to get $8x = 40$, or $x = 5$. Substitute $x = 5$ into either equation to get $y = 4$. The solution is $(5, 4)$.

Solve each system using any method. Explain why you chose the method you used.

32. $y = 2.5x$
 $2y + 3x = 32$

32. (4, 10); Answers may vary. Sample: substitution; the first equation is already solved for y.

33. $2x + y = 4$
 $6x + 7y = 12$

33. (2, 0); Answers may vary. Sample: substitution; the first equation is easily solved for y.

34. $3x + 2y = 5$
 $4x + 5y = 16$

34. (-1, 4); Answers may vary. Sample: elimination; neither equation solves easily for x or y.

Solve each system using any method. Explain why you chose the method you used.

35. $y = \frac{2}{3}x + 1$

$$2x + 3y = 27$$

35. (6, 5); Answers may vary. Sample: substitution; the first equation is already solved for y .


36. $x + y = 1.5$

$$2x + y = 1$$

36. (-0.5, 2); Answers may vary. Sample: elimination; you can subtract the equations as they are.

37. $\frac{1}{3}x + \frac{1}{2}y = 0$
 $\frac{1}{2}x + \frac{1}{5}y = \frac{11}{5}$

37. (6, -4); Answers may vary. Sample: elimination; you can multiply each equation by the LCD of the denominators to eliminate the fractions. Then you can use elimination.

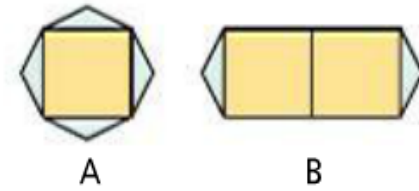
-  **38. Compare and Contrast** What do the substitution method and the elimination method have in common? Explain. Give an example of a system that you would prefer to solve using one method instead of the other. Justify your choice.

38. They both result in an equation with a single variable to solve.

39. Vacations A hotel offers two activity packages. One costs \$192 and includes 3 h of horseback riding and 2 h of parasailing. The second costs \$213 and includes 2 h of horseback riding and 3 h of parasailing. What is the cost for 1 h of each activity?

39. parasailing: \$51; horseback riding: \$30

40. **Geometry** Each of the squares in the figures shown at the right has the same area, and each of the triangles has the same area. The total area of Figure A is 141 cm^2 . The total area of Figure B is 192 cm^2 . What is the area of each square and each triangle?



40. square: 81 cm^2 ; triangle: 15 cm^2

**Challenge** Solve each system using elimination.

$$\begin{aligned} 41. \quad \frac{2}{x} - \frac{3}{y} &= -5 \\ \frac{4}{x} + \frac{6}{y} &= 14 \end{aligned}$$

$$41. \quad \left(2, \frac{1}{2}\right)$$

$$\begin{aligned} 42. \quad 2x &= 5(2 - y) \\ y &= 3(-x + 5) \end{aligned}$$

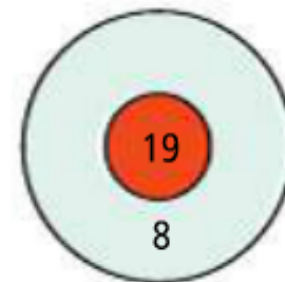
$$42. \quad (5, 0)$$

$$\begin{aligned} 43. \quad 2x - 3y + z &= 0 \\ 2x + y + z &= 12 \\ y - z &= 4 \end{aligned}$$

$$43. \quad (5, 3, -1)$$



44. Reasoning Use the dartboard at the right. Can you score exactly 100 points with seven darts that all land on the board? Explain.



44. Yes; the problem can be represented by the system

$$x + y = 7$$

$$19x + 8y = 100$$

You can score 100 with four 19's and three 8's.

Standardized Test Prep

45. What is the value of the y -coordinate of the solution of the given system?

$$4x + 3y = 33$$

$$3x + 2y = 23$$

45. 7

46. What is the y -intercept of $2x + 5y = 15$?

46. 3

47. You buy a toothbrush for \$2.83 and a tube of toothpaste for \$2.37. There is a 5% sales tax. Including the tax, what is the total cost in dollars of your purchases?

47. 5.46

- 48.** Three fire trucks and 4 ambulances can fit into a parking lane 152 ft long. Two fire trucks and 5 ambulances can fit into a lane 136 ft long. How many feet long must a parking lane be for 1 fire truck and 5 ambulances? Assume there is 1 ft of space between each vehicle.



48. 105

- 49.** You are competing in a mountain bike race. Your average speed is 10 mi/h. If the racecourse is 65 mi long, how many minutes will it take you to finish the race?

49. 390

Mixed Review

Solve each system using substitution.

$$\begin{aligned} 50. \quad y &= \frac{1}{2}x \\ 2y + 3x &= 28 \end{aligned}$$

$$\mathbf{50. (7, 3.5)}$$

$$\begin{aligned} 51. \quad x - 7 &= y \\ 2x - y &= 41 \end{aligned}$$

$$\mathbf{51. (34, 27)}$$

$$\begin{aligned} 52. \quad x + 2y &= -1 \\ 3x - 5y &= 30 \end{aligned}$$

$$\mathbf{52. (5, -3)}$$

Solve each inequality.

53. $4 - 2a < 3a - 1$

53. $a > 1$

54. $3(2x - 1) \geq 5x + 4$

54. $x \geq 7$

55. $2.7 + 2b > 3.4 - 1.5b$

55. $b > 0.2$

Get Ready! To prepare for Lesson 6-4, do Exercise 56.

56. Two trains run on two sets of parallel tracks. The first train leaves a city $\frac{1}{2}$ h before the second train. The first train travels at 55 mi/h. The second train travels at 65 mi/h. How long does it take for the second train to pass the first train?

56. 2.75 h

End 6-3