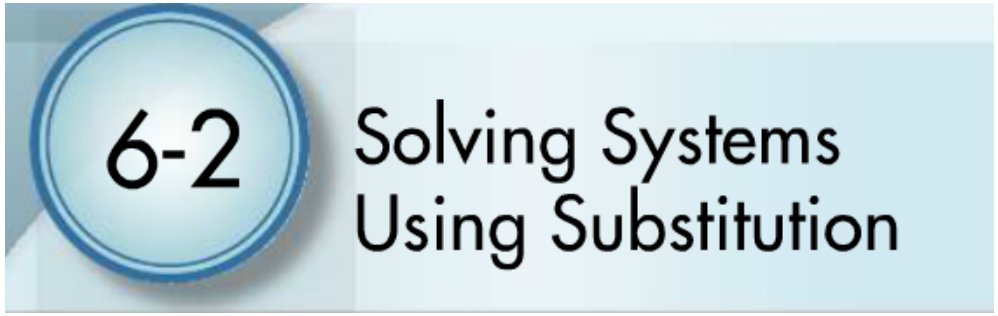


PURPOSE To use substitution to solve a real-world situation that can be represented by a system of linear equations

Lesson Check

Do you know HOW?

Solve each system using substitution. Check your solution.

1. $4y = x$
   $3x - y = 70$

2. $-2x + 5y = 19$
   $3x - 4 = y$

Tell whether the system has *one solution*, *infinitely many solutions*, or *no solution*.

3. $y = 2x + 1$
   $4x - 2y = 6$

4. $-x + \frac{1}{2}y = 13$
   $x + 15 = \frac{1}{2}y$

5. **Talent Show** In a talent show of singing and comedy acts, singing acts are 5 min long and comedy acts are 3 min long. The show has 12 acts and lasts 50 min. How many singing acts and how many comedy acts are in the show?

Do you UNDERSTAND? MATHEMATICAL PRACTICES

6. **Vocabulary** When is the substitution method a better method than graphing for solving a system of linear equations?

For each system, tell which equation you would first use to solve for a variable in the first step of the substitution method. Explain your choice.

7. $-2x + y = -1$
   $4x + 2y = 12$

8. $2.5x - 7y = 7.5$
   $6x - y = 1$

Tell whether each statement is *true* or *false*. Explain.

9. When solving a system using substitution, if you obtain an identity, then the system has no solution.

10. You cannot use substitution to solve a system that does not have a variable with a coefficient of 1 or -1.

1. $\left(25\frac{5}{11}, 6\frac{4}{11}\right)$ **2.** (3, 5)
3. no solution **4.** no solution
5. 7 singing, 5 comedy

6. Answers may vary. Sample: Graphing a system can be inexact, and it is very difficult to read the intersection, especially when there are noninteger solutions. The substitution method is better, as it can always give an exact answer.

7. $-2x + y = -1$ because it is easily solved for y.
8. $6x - y = 1$ because it is easily solved for y.
9. False; it has infinitely many solutions.
10. False; you can use it, but the arithmetic may be harder.

Practice and Problem-Solving Exercises

MATHEMATICAL PRACTICES

A Practice

Solve each system using substitution. Check your answer.

Solve each system using substitution. Check your answer.

11. $x + y = 8$
$y = 3x$

11. (2, 6)

12. $2x + 2y = 38$
$y = x + 3$

12. (8, 11)

13. $x + 3 = y$
$3x + 4y = 7$

13. $\left(-\frac{5}{7}, 2\frac{2}{7}\right)$

Solve each system using substitution. Check your answer.

14. $y = 8 - x$
$7 = 2 - y$

14. (13, −5)

15. $y = -2x + 6$
$3y - x + 3 = 0$

15. (3, 0)

16. $3x + 2y = 23$
$\frac{1}{2}x - 4 = y$

16. $\left(7\frac{3}{4}, -\frac{1}{8}\right)$

Solve each system using substitution. Check your answer.

17. $y - 2x = 3$
$3x - 2y = 5$

17. $(-11, -19)$

18. $4x = 3y - 2$
$18 = 3x + y$

18. $(4, 6)$

19. $2 = 2y - x$
$23 = 5y - 4x$

19. $(-12, -5)$

Solve each system using substitution. Check your answer.

20. $4y + 3 = 3y + x$
$2x + 4y = 18$

20. (5, 2)

21. $7x - 2y = 1$
$2y = x - 1$

21. $\left(0, -\frac{1}{2}\right)$

22. $4y - x = 5 + 2y$
$3x + 7y = 24$

22. (1, 3)

23. **Theater Tickets** Adult tickets to a play cost $22. Tickets for children cost $15. Tickets for a group of 11 people cost a total of $228. Write and solve a system of equations to find how many children and how many adults were in the group.

23. 2 children, 9 adults

24. Transportation A school is planning a field trip for 142 people. The trip will use six drivers and two types of vehicles: buses and vans. A bus can seat 51 passengers. A van can seat 10 passengers. Write and solve a system of equations to find how many buses and how many vans will be needed.

24. 2 buses, 4 vans

25. **Geometry** The measure of one acute angle in a right triangle is four times the measure of the other acute angle. Write and solve a system of equations to find the measures of the acute angles.

25. 18°, 72°

Tell whether the system has *one solution, infinitely many solutions,* or *no solution.*

26. $y = \frac{1}{2}x + 3$
$2y - x = 6$

27. $6y = -5x + 24$
$2.5x + 3y = 12$

28. $x = -7y + 34$
$x + 7y = 32$

26. Infinitely many solutions

27. Infinitely many solutions

28. No Solution

Tell whether the system has *one solution, infinitely many solutions,* or *no solution.*

29. $5 = \frac{1}{2}x + 3y$
$10 - x = 6y$

29. Infinitely many solutions

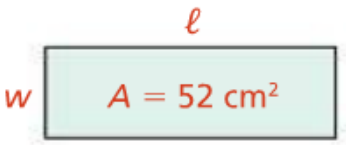
30. $17 = 11y + 12x$
$12x + 11y = 14$

30. No Solution

31. $1.5x + 2y = 11$
$3x + 6y = 22$

31. One Solution

32. **Geometry** The rectangle shown has a perimeter of 34 cm and the given area. Its length is 5 more than twice its width. Write and solve a system of equations to find the dimensions of the rectangle.



32. $2\ell + 2w = 34$
$\ell = 2w + 5$
4 cm by 13 cm

33. Writing What would your first step be in solving the system below? Explain.

$$1.2x + y = 2$$
$$1.4y = 2.8x + 1$$

33. Solve $1.2x + y = 2$ for y because then you can solve the system using substitution.

34. Coins You have $3.70 in dimes and quarters. You have 5 more quarters than dimes. How many of each type of coin do you have?

34. 7 dimes and 12 quarters

35. Error Analysis Describe and correct the error at the right in finding the solution of the following system:

$7x + 5y = 14$
$x + 8y = 21$

Step 1 $x + 8y = 21$
$x = 21 - 8y$

Step 2 $x + 8y = 21$
$(21 - 8y) + 8y = 21$
$21 = 21$

The system has infinitely many solutions.

35. The student solved an equation for x but then substituted it into the same equation, not the other equation.

$x + 8y = 21$, so $x = 21 - 8y$

$$7(21 - 8y) + 5y = 14$$
$$147 - 56y + 5y = 14$$
$$-51y = -133$$
$$y = \frac{-133}{-51} = 2\frac{31}{51}$$

So,

$$x = 21 - 8\left(2\frac{31}{51}\right) = 21 - \frac{1064}{51} = \frac{7}{51}$$

The solution is $\left(\frac{7}{51}, 2\frac{31}{51}\right)$.

36. Art An artist is going to sell two sizes of prints at an art fair. The artist will charge $20 for a small print and $45 for a large print. The artist would like to sell twice as many small prints as large prints. The booth the artist is renting for the day costs $510. How many of each size print must the artist sell in order to break even at the fair?

36. 6 large, 12 small

37. Think About a Plan At a certain high school, 350 students are taking an algebra course. The ratio of boys to girls taking algebra is 33 : 37. How many more girls are taking algebra than boys?

- How can you write a system of equations to model the situation?
- Which equation will you solve for a variable in the first step of solving the system? Why?
- How can you interpret the solution in the context of the problem?

37. 20 more girls

38. a. Compare and Contrast Using a graph, how can you tell when a system of linear equations has no solution?

b. Using substitution, how can you tell when a system of linear equations has no solution?

c. How can you tell by looking at a table of values if two lines will intersect in one point, no points, or an infinite number of points?

38. a. The lines will be parallel.

b. A false statement results.

c. Compare the tables for both equations using the same *x*-values in both tables. If one of the *x*-values has the same *y*-values in both tables, the lines will intersect at one point. If the tables are exactly the same, the lines will intersect at an infinite number of points. If you can add a constant to the y-values in one table to get the *y*-values in the second table, the lines will not intersect.

39. Fireworks A pyrotechnician plans for two fireworks to explode together at the same height in the air. They travel at speeds shown at the right. Firework B is launched 0.25 s before Firework A. How many seconds after Firework B launches will both fireworks explode?

39. 2.75 s

40. Writing Let a be any real number. Will the system at the right *always*, *sometimes*, or *never* have a solution? Explain.

$$y = ax$$
$$y = ax + 4$$

40. Never; the lines are parallel.

41. Reasoning Explain how you can use substitution to show that the system at the right has no solution.

$$y + x = x$$
$$\frac{3x}{2y} = 4$$

41. Answers may vary. Sample: Solve the first equation, $y + x = x$, for y, so $y = x - x = 0$. But the second equation is not defined for $y = 0$; therefore, there is no solution.

Challenge

42. Agriculture A farmer grows corn, tomatoes, and sunflowers on a 320-acre farm. This year, the farmer wants to plant twice as many acres of tomatoes as acres of sunflowers. The farmer also wants to plant 40 more acres of corn than of tomatoes. How many acres of each crop should the farmer plant?

42. corn: 152 acres; tomatoes: 112 acres; sunflowers: 56 acres

43. Track and Field Michelle and Pam are running a 200-m race. Michelle runs at an average of 7.5 m/s. Pam averages 7.8 m/s, but she starts 1 s after Michelle.

a. How long will it take Pam to catch up to Michelle?

b. Will Pam overtake Michelle before the finish line? Explain.

43. a. 25 s after Pam starts, 26 s after Michelle starts

b. Yes; 26 s after Michelle starts, both runners will be at 195 m. Pam, who is running at a faster rate, will go on to win.

44. What is the value of the x-coordinate of the solution of the given system?

$$2x + 3y = 144$$
$$y - x = 24$$

44. 14.4

45. You are making blueberry muffins and need to buy a muffin tin and baking cups. Each package of baking cups has 50 baking cups and costs \$1.25. The muffin tin costs \$15. If you have \$22 to spend, at most how many baking cups can you buy?

45. 250

Standardized Test Prep

46. What is the x-intercept of $2y - 3x = 24$?

46. -8

47. An online store charges 4% of the cost of an order to cover shipping costs. How much would you pay in dollars for shipping on an order that costs \$146?

47. \$5.84

48. What is the solution of the equation $2x - 3 = 8$?

48. 5.5

Solve each system by graphing. Tell whether the system has *one solution*, *infinitely many solutions*, or *no solution*.

49. $y = 3x + 3$
$y = x - 3$

49. one solution:

$(-3, -6)$

50. $y = x + 1$
$2x + y = 10$

50. one solution:

$(3, 4)$

51. $y = -x + 2$
$x + y = 3$

51. no solution

Find the slope of a line perpendicular to the graph of each equation.

52. $y = 3x$

52. $-\frac{1}{3}$

53. $y = -\frac{1}{4}x$

53. 4

54. $\frac{1}{3}x - y = 2$

54. -3

Get Ready! **To prepare for Lesson 6-3, do Exercises 55–57.**

Solve each equation. Check your answer.

55. $5x + 1 = 3x - 5$

55. -3

56. $4c - 7 = -c + 3$

56. 2

57. $5k + 7 = 3k + 10$

57. $\frac{3}{2}$

End 6-2